Multiphase flows

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CONDENSATION

Definition: Condensation is the heat transfer process by which a standard vapor is converted into a liquid by means of removing the latent heat of condensation.

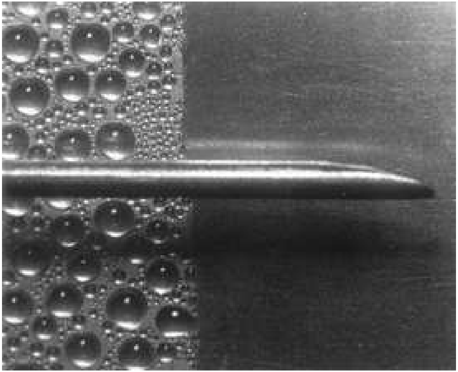
Types of condensation

- Outside of surfaces
  - Drop-wise
  - Direct contact
  - Homogeneous
  - Film-wise
- Inside tubes

References:

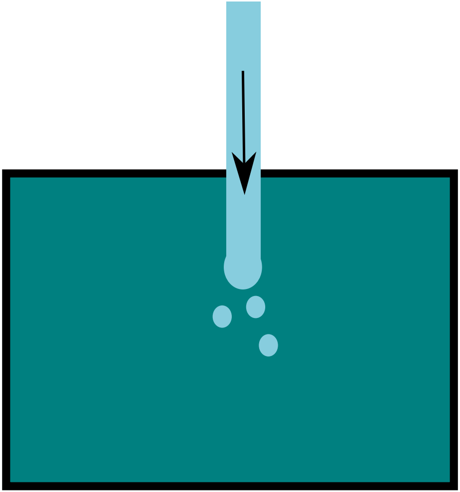
J.R Thome Wolverine. Engineering Data Book III.
C.E. Brennen. Fundamentals of multiphase flow. Cambridge. University Press. 2005 (http://authors.library.caltech.edu/25021/). Chapter 6
Perry's Chemical Engineering Handbook (McGraw-Hill Handbooks): Chapter 5-20.

Drop-wise condensation



Positive: Very high heat transfer coefficients.
Negative: Special materials, difficult to control in industrial processes

Direct contact condensation

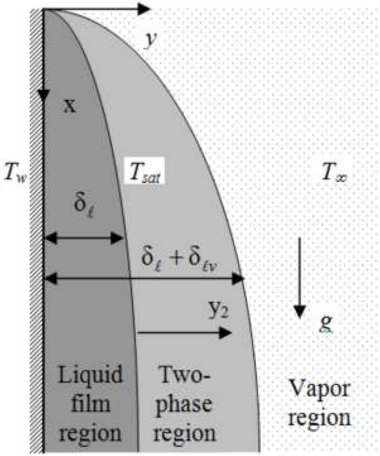


Positive: Very efficient.
Negative: Condensate and coolant are difficult to separate.

Homogeneous condensation

Positive: ?.
Negative: It is usually an undesirable effect.

Film condensation



Large industrial interest.

It was one of the first problems successfully analyzed from a fundamental point of view by **Nusselt** (1916).

Film condensation:

- 1916: Nusselt develops a solution from the fundamental point of view (see exercise)
- 1959: Sparrow and Greg include boundary layer analysis in the development
- 1962: Bromley extends the theory to include the effects of subcooling in the condensate

The only relevant differences are observed for $\mathrm{Pr} \ll 1$ (water $Pr \approx 0.72$).

How to obtain the heat exchange coefficient (h)?

$$q = h\Delta T \quad [W/m^2] \tag{1}$$

We need to find out q and to write it as a function of ΔT. q is given by the fourier laws:

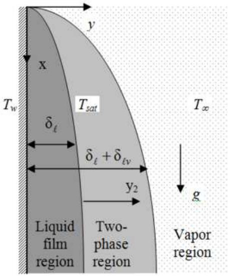
$$q = -\kappa \nabla T \tag{2}$$

in 1D

$$q = -\kappa \frac{\partial T}{\partial x} \tag{3}$$

We need to find out the function $T(x)$. How? Solving for the energy equation:

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p u \frac{\partial T}{\partial x} = \kappa_l \frac{\partial^2 T}{\partial x^2} \tag{4}$$



$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p u \frac{\partial T}{\partial x} = \kappa_l \frac{\partial^2 T}{\partial x^2} \quad (5)$$

Steady state + negligible advection term

$$0 = \kappa_l \frac{\partial^2 T}{\partial x^2} \quad (6)$$

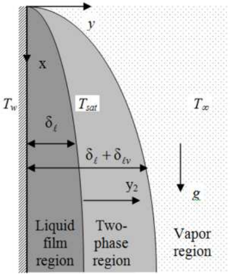
The solution is:

$$T(x) = T_{wall} + \frac{T_{int} - T_{wall}}{\delta} x \quad (7)$$

and therefore the heat flux is

$$q = -\kappa \frac{\partial T}{\partial x} = \kappa \frac{\Delta T}{\delta} \quad (8)$$

Ok, the problem now is to find out $\delta(x)$



Momentum equation in x:

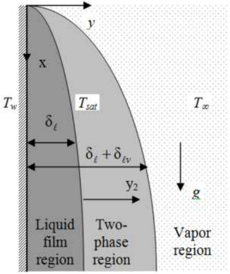
$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x}+\rho g+\mu_l\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (9)$$

Momentum equation in y:

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y}+\mu_l\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (10)$$

Simplifications:

- No forces at the interface $\rightarrow \frac{\partial u}{\partial y} \approx 0$
- Gravity in x
- The vaporization energy is more important that the energy exchanged by conduction
- The variations in the speed in x are negligible $\left(\frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2}\right)$
- Gas at rest (u=0)
- Steady state



Momentum equation in x (for liquid):

$$0 = -\frac{\partial p_l}{\partial x} + \rho_l g + \mu_l \frac{\partial^2 u}{\partial y^2} \tag{11}$$

Momentum equation in x (for gas/vapor where u=0):

$$0 = -\frac{\partial p_v}{\partial x} + \rho_v g \tag{12}$$

Momentum equation in y (liquid):

$$0 = -\frac{\partial p_l}{\partial y} \tag{13}$$

Momentum equation in y (vapor):

$$0 = -\frac{\partial p_v}{\partial y} \tag{14}$$

Momentum equation at the interface

$$p_l = p_v + \frac{2\sigma}{R_c} \tag{15}$$

Conclusions:

- From the momentum at the interface we obtain that as the interface is plane (radius of curvature $R_c \to \infty$) the pressure at both sides of the interface is the same $p_l = p_v$
- From the momentum in y we obtain that p_l and p_v are constant in y-direction. As they are equal at the interface we find ($p_l = p_v = p$)
- From the momentum equation in x for the vapor we know that

$$\frac{\partial p}{\partial x} = \rho_v g \tag{16}$$

- Replacing in the momenum in x for the liquid we find

$$\mu_l \frac{\partial^2 u}{\partial y^2} = -g(\rho_l - \rho_v) \tag{17}$$

To solve

$$\mu_l \frac{\partial^2 u_l}{\partial y^2} = -g(\rho_l - \rho_v) \tag{18}$$

we need to apply boundary conditions:

- At $y = 0$, $u_l = 0$
- At $y = \delta$, $\frac{\partial u_l}{\partial y}$ = 0

Sol:

$$u(y) = \frac{g(\rho_l - \rho_v)}{\mu_v} \delta^2 \left[\frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^2 \right] \tag{19}$$

Then the mass flow is

$$\dot{m} = \rho_l \int u(y) dS = \rho_l \int_0^\delta u(y) b dy = \frac{\rho_l b (\rho_l - \rho_v) g \delta^3}{3 \mu_l} \tag{20}$$

Normally, we define

$$\Gamma = \frac{\dot{m}}{b} \tag{21}$$

Energy balance

- We have seen that the heat flux is given by

$$q = \kappa_l \frac{\Delta T}{\delta} \tag{22}$$

- This is the flux that must the released by the condensation

$$q_{cond} = \frac{\Delta H_{vap} \dot{m}}{bdx} \tag{23}$$

- Balancing both terms we obtain ($q_{cond} = q$)

$$\frac{d\Gamma}{dx} = \frac{\kappa_l \Delta T}{\Delta H_{vap} \delta} \tag{24}$$

and using the expression for Γ obtained before

$$\delta^3 d\delta = \frac{\kappa_l \mu_l \Delta T}{\rho_l g(\rho_l - \rho_v)\Delta H_{vap}} dx \tag{25}$$

Integrating from 0 to δ and from 0 to x, we obtain

$$\delta(x) = \left[\frac{4\mu_l \kappa_l \Delta T}{g\rho_l(\rho_l - \rho_v)\Delta H_{vap}} x\right]^{1/4} \tag{26}$$

Now we can find out the heat transfer coefficient

$$q = h\Delta T = \frac{\kappa_l \Delta T}{\delta} \rightarrow h = \frac{\kappa_l}{\delta} \tag{27}$$

This heat transfer coefficient is local (depends on x). For that reason, we average over all the length of the film

$$\overline{h} = \frac{\int_0^L h dx}{L} = 0.943 \left[\frac{g\rho_l(\rho_l - \rho_v)\Delta H_{vap}\kappa_l^3}{\mu_l \kappa_l \Delta T} \right]^{1/4} \tag{28}$$

Interfacial phenomena:

- At $Re_\Gamma = \dfrac{4\Gamma}{\mu_L} \approx 30$ we start to observe waves at the interface
- At $Re_\Gamma \approx 2000$ the film becomes turbulent

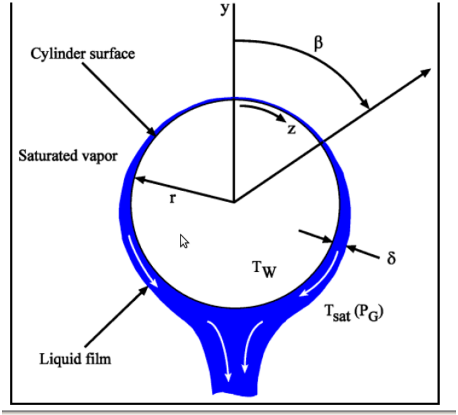
Wave effect:

- Waves increase the exchange surface
- The boundary layer thickness is stretched

Kutateladze proposes this modification for the averaged heat transfer coefficient

$$\frac{h}{\kappa_l}\left[\frac{\mu_l^2}{\rho_l(\rho_l-\rho_g)g}\right]^{1/3} = \frac{Re_\Gamma}{1.08Re_\Gamma^{1.22}-5.2} \tag{29}$$

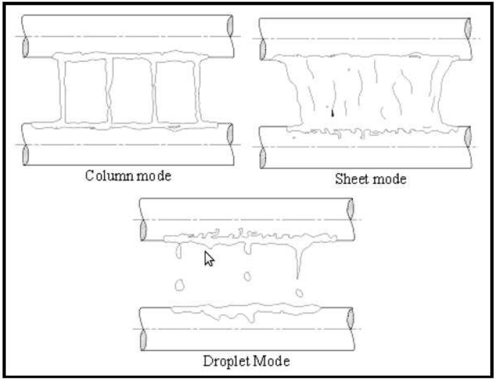
In heat exchanger we typically use tubes



It is possible to derive analytical expressions for tubes (see problem)

$$Nu_D = \frac{hD}{\kappa_L} = 0.728 \left(\frac{Ra}{Ja}\right)^{1/4} \quad (30)$$

Finally, once we derive the flow around one tube, we have to consider that we have a row of tubes:



The final expression of Nth tubes on a row:

$$\frac{h(ND)}{\kappa_l} = 0.728 \left[\frac{g(\rho_l - \rho_v)(ND)^3 \Delta H_{vap}}{\kappa_l \nu_l \Delta T} \right]^{1/4} \tag{31}$$

or

$$\frac{\alpha}{\alpha(N=1)} = N^{-1/4} \tag{32}$$

In practice, Kern found in 1958 that

$$\frac{\alpha}{\alpha(N=1)} = N^{-1/6} \tag{33}$$