Multiphase flows

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EVAPORATION

Definition: Evaporation is the process by which a standard liquid is converted into vapor by means of supplying the latent heat of evaporation.

Evaporation

- Boiling
- Cavitation

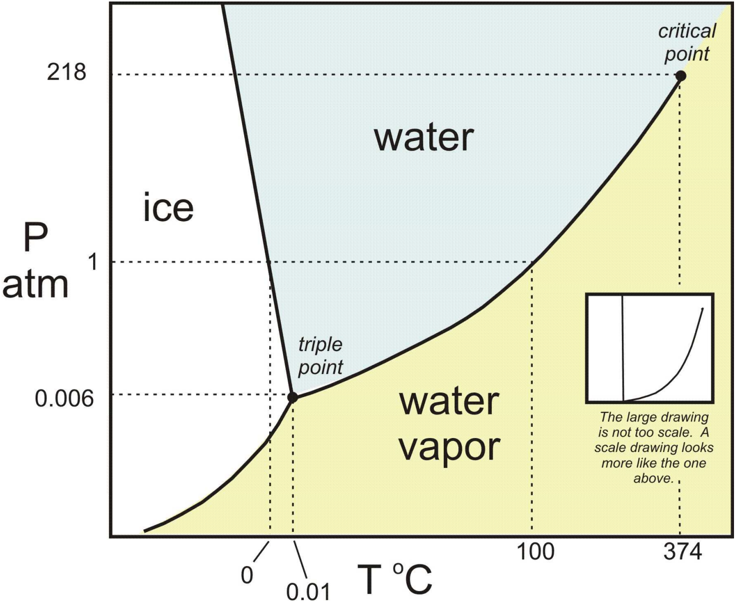
References:

C.E. Brennen. Fundamentals of multiphase flow. Cambridge. University Press. 2005 (http://authors.library.caltech.edu/25021/). Chapter 4,5,6,7

Perry's Chemical Engineering Handbook (McGraw-Hill Handbooks): Chapters 5-20.

C.E. Brennen. Cavitation and bubble dynamics. Oxford University Press. 1995 (http://authors.library.caltech.edu/25017/) Chapter 1,2,4

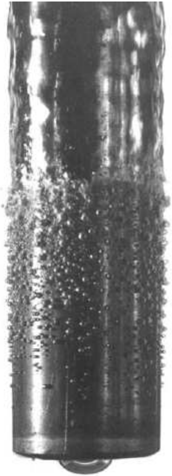
Phase diagrams

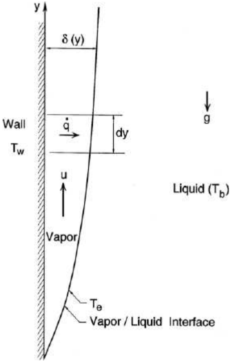


Boiling: The mechanism responsible of the phase change is heating.
Some types

- Film boiling
- Pool boiling
  - Subcooled pool boiling
  - Saturated pool boiling

Film boiling: Similar to film condensation

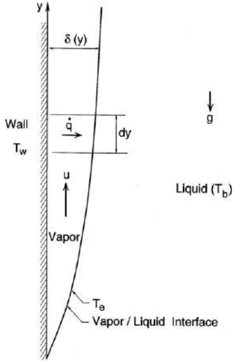


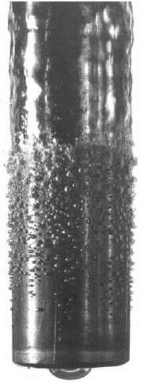


Film boiling: Similar to film condensation

$$\delta(y) = \left[\frac{4\kappa_v \Delta T \mu_v}{3\rho_v(\rho_l - \rho_v) g \Gamma}\right]^{1/4} y^{1/4}$$

$$h = \left[\frac{3\rho_v(\rho_l - \rho_v) g \Gamma \kappa_v^3}{4\Delta T \mu_v}\right]^{1/4} y^{-1/4}$$





We can use evaporation to:

- Evaporate the liquid
- Refrigerate the tube

The liquid is able to evacuate larger amount of heat than the gas ($\kappa_l \gg \kappa_v$).

When there is subcooled evaporation the heat transfer is enhanced
however...
when vapor covers all the tube's surface we can overheat the tube and to induce failure.

Bubble nucleation: It is the process by which bubbles appear in the liquid

Types:

- Homogeneous nucleation: We need to concentrate the energy at a given point in order to break the inter-molecular forces
- Heterogeneous nucleation: We need to make small bubble nuclei grow

Homogeneous nucleation
We need to calculate the energy to create one bubble.

Initial state: Pure liquid.
Final state: Vapor bubble + liquid.

We take as reference the initial state ($E_0 = 0$).
The final energy of the system is accumulated at the interface, its expression is

$$E_f = 4\pi R_b^2 \sigma \tag{1}$$

Part of this energy is externally supplied when as the work required we displace the liquid to create one bubble

$$W_d = \frac{4}{3}\pi R_b^3 \Delta p \tag{2}$$

the other part of the energy must be supplied by the liquid.

$$W_{required} = 4\pi R_b^2 \sigma - \frac{4}{3}\pi R_b^3 \Delta p \tag{3}$$

Δp is the pressure jump at the bubble inteface, which can be obtained from the Laplace eq (momentum equation applied at the interface)

$$p_{gas} + p_{vap} = 2\frac{\sigma}{R_b} + p_l \tag{4}$$

As we assume pure liquid ($p_{gas} = 0$), then the Laplace equation gives

$$\Delta p = p_{vap} - p_l = 2\frac{\sigma}{R_b} \tag{5}$$

Thus

$$W_{required} = \frac{4}{3}\pi R_b^2 \sigma \tag{6}$$

This energy is obtained from the energy fluctuations from the molecules, which is proportional to the temperature $E_{molecules} \sim k_{boltz} T$.

For that reason, the most important parameter in homogeneous nucleation is the Gibbs number

$$Gb = \frac{W_{required}}{k_{boltz} T} \tag{7}$$

There are different theories to predict the number of bubbles generated per unit time and volume (J) as

$$J = J_0 \exp\left(-Gb\right) \qquad (8)$$

Blander & Katz propse

$$J_0 = N \left(\frac{2\sigma}{\pi M}\right)^{1/2} \qquad (9)$$

N: molecules/vol.

- Note that we predict bubbles appearing at all temperatures, the only different is the amount of bubbles generated.
- Also note that if we have water at 1 atm and 25 degrees Celsius, the bubble will be formed but it will condense inmediately

Heterogeneous nucleation

In heterogeneous nucleation we assume that there are small gas bubbles already contained in the liquid (most of the time they stick to walls).

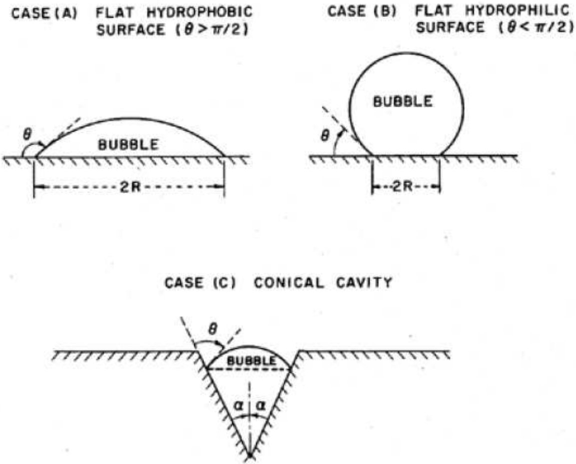


Figure 1.6: Various modes of heterogeneous nucleation.

Theory:
We can obtain what is the equilibrium pressure as a function of the bubble radius

$$p_{gas} + p_{vap} = 2\frac{\sigma}{R_b} + p_l \tag{10}$$

If we look at cavitation processes, T is constant the therefore

$$pV = Cte \tag{11}$$

That is

$$pR^3 = p_0 R_0^3 \tag{12}$$

p_{vap} is only a function of the temperature, therefore

$$p_l = p_0 \left(\frac{R_0}{R}\right)^3 + p_{vap} - 2\frac{\sigma}{R_b} \tag{13}$$

We have $p_l(R)$, that is, given an initial bubble nuclei, we can find the pressure of the system in equilibrium when this bubble expands

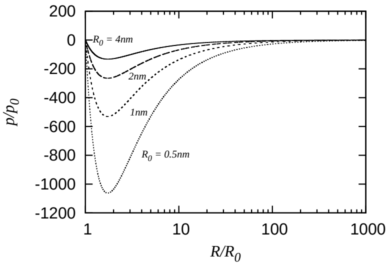


Figure: There is a minimum pressure below which the bubble cannot stay in equilibrium with the liquid and it grows

We can do the same for the temperature

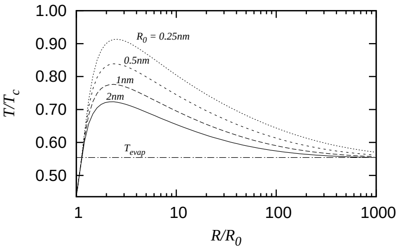
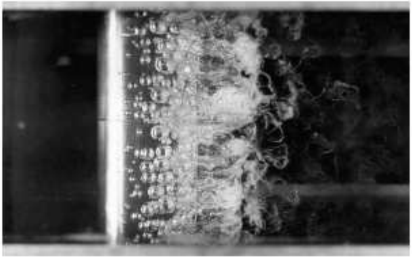
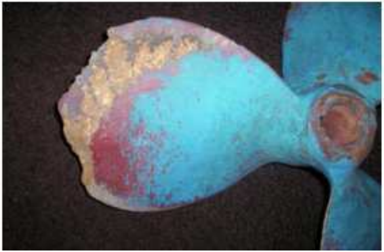


Figure: (a)Evolution of the bubble radius and the water temperature as a function of the initial bubble radius.

Bubble dynamics:





To investigate the dynamics of bubbles is very interesting in order to understand the behavior of bubble systems.

The equation that governs the dynamics of the bubble is called Rayleigh Plesset equation.

- Problem: To obtain how the bubble radius changes when the pressure far from the bubble is modified.
- Phases = 2.
- Equations:
  - Continuity, momentum and energy equations inside the bubble
  - Continuity, momentum and energy equations in the liquid
  - Continuity, momentum and energy equations at the interface

Simplifications:

- We assume that the bubble is only gas (no vapor and no mass transfer across the interface). Thus, the mass balance inside the bubble reduces to

$$m_{gas} = Cte \tag{14}$$

that is

$$p_{gas} = f(R_b, T) \tag{15}$$

Typically we can use an adiabatic or isothermal model to find out the pressure as a function of the volume.

- As there is no mass transfer (J=0), the continuity equation at the interface says

$$u_l(r = R) = u_b(r = R) = \dot{R} \tag{16}$$

- We assume that there is only radial motion. The continuity equation in spherical coordinates in the liquid says:

$$div \cdot u_l = \frac{1}{r^2}\frac{\partial r^2 u_l}{\partial r^2} = 0 \tag{17}$$

That is

$$r^2 u_l = Cte = R^2 \dot{R} \tag{18}$$

The velocity in any arbitrary point of the liquid can be expressed as a function of the radius and velocity of the bubble interface as

$$u(r) = \dot{R}R^2 r^{-2} \tag{19}$$

The momentum equation in the liquid is

$$\rho_l \frac{\partial u_l}{\partial t} + \rho_l u_l \frac{\partial u_l}{\partial r} = -\frac{\partial p_l}{\partial r} + \frac{\mu_l}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u_l}{\partial r}\right) \tag{20}$$

We know that

- $r = R$, $u(r) = \dot{R}$
- $r = \infty$, $p_l = p_\infty$

We can also relate the pressure of the liquid at the interface with the pressure inside the bubble using the momentum equation applied to the interface

$$p_l(r = R) = p_b - 2\frac{\sigma}{R_b} \tag{21}$$

Replacing $u(r)$ obtained from the continuity equation and applying the boundary conditions we obtain the following equation

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{p_\infty - p_l(r = R)}{\rho_l} \tag{22}$$

The Rayleigh-Plesset equation give us a lot of information about the behavior of a single bubble.

For example, if we increase the pressure instantaneously we can obtain expressions for the characteristic implosion time and minimum radius

$$t_{imp} = 0.915 \left(\frac{\rho_l R_0}{p_\infty - p_0} \right)^{1/2} \tag{23}$$

$$R_{min} = R_0 \left[\frac{1}{k-1} \frac{p_{g0}}{p_\infty - p_0 + \frac{3\sigma}{R_0}} \right]^{\frac{1}{3(k-1)}} \tag{24}$$

where k is the polytropic coefficient (k=1 for isothermal collapse and k=$\frac{c_p}{c_v}$ for an adiabatic implosion.

We can find out the bubble resonance frequency.

We assume that the pressure variation at infinity obeys

$$p_\infty(t) = p_{ref} + \Delta p cos(\omega t) \tag{25}$$

This is equivalent to

$$p_\infty(t) = p_{ref} + Real(\Delta p \exp{(i\omega t)}) \tag{26}$$

where

$$\exp{i\omega t} = cos(\omega t) + isin(\omega t) \tag{27}$$

Now we want to know the response of the bubble. In the linear regime, (weak oscillations), we know the bubble radius is also a sinusoidal function, however, we do not know the amplitude and the lag. Mathematically we say that we want to find

$$R = R_0 + R_0 Real(\psi \exp{(i\omega t)}) \tag{28}$$

Note that ψ is a complex number that will allow us to obtain the amplitude and the phase of the bubble radius oscillation.

From the equation of the radius, we can obtain expressions for $\dot{R}$ and $\ddot{R}$

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{p_\infty - p_l(r = R)}{\rho_l} \tag{29}$$

Substituting in the RP equation we find an equation as a function of ω and ψ. The frequency for which we obtain the largest amplitude of the oscillation for a given Δp is called bubble resonant frequency

$$\omega_p \left(\frac{3k(p_{ref} - p_v)}{\rho_l R_0^2} + \frac{2(3k-1)\sigma}{\rho_l R_0^3} - \frac{8\nu_l^2}{R_0^4} \right)^{1/2} \tag{30}$$

The *bubble natural frequency* is defined when there is no dumping ($\nu_l = 0$).

Exercise: Discuss how the bubble behaves below, above and for values around the bubble natural frequency.