Multiphase flows

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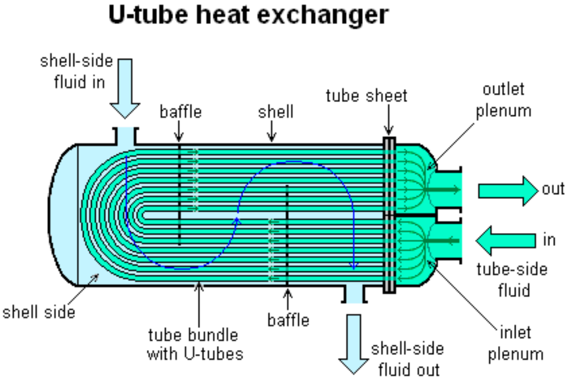
- Models
- Types of multiphase flow

In the previous class we saw the basic equations that we can apply to global systems



Global balance give us a lot of information (do example vaporization 100%)

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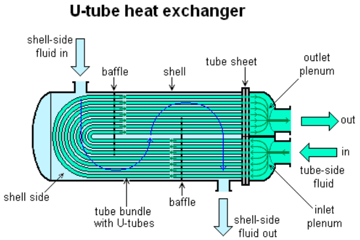


It provides information about what it happens outside,

- Required inlet pressure, heat exchange, input/output temperatures,....

but not inside

- Exchange area?
- All types of flow are the same?

The same reasoning applies to other systems like engines (quality of the combustion), refrigeration towers and other systems.

Global balance give us information about global behavior, if we want to know details about the equipment (improvements) we need to look inside

We can try to understand what happens inside applying more complex models

- Homogeneous models
- Heterogeneous models

The model that has to be used depends on the type of flow

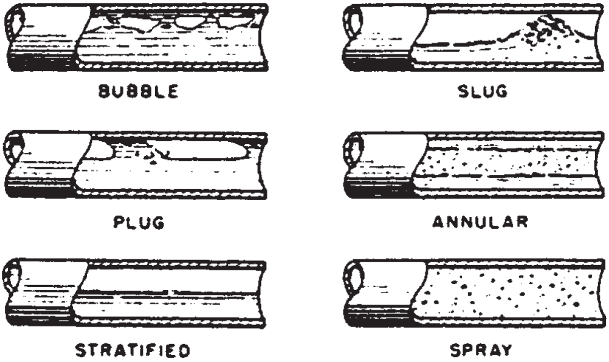


Figure: Flow types in horizontal tubes

Homogeneous models: We consider a single effective fluid that is the averaged of both phases (example: heat transfer coefficients around multiphase flows)

Example: Heat transfer (1D-Energy equation)
Full problem (no model). We use the energy equation to obtain the temperature profile:

$$\rho_f c_{p,f} \frac{\partial T}{\partial t} + \rho c_p u \frac{\partial T}{\partial x} = \kappa_f \frac{\partial^2 T}{\partial x^2} \quad (1)$$

For a solid:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \quad (2)$$

κ: Thermal conductivity
a: Thermal diffusivity. For liquids

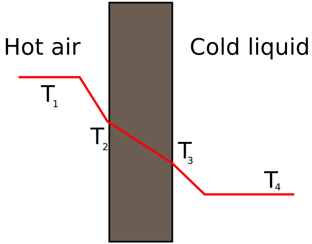
$$a = \frac{\lambda}{c_p \rho} \quad (3)$$

Once the temperature is obtained, we can use Fourier's law to obtain the heat flux

$$q = -\kappa \nabla T = -\kappa \frac{\partial T}{\partial x} \quad (4)$$

$$\rho_f c_{p,f} \frac{\partial T}{\partial t} + \rho c_p u \frac{\partial T}{\partial x} = \kappa_f \frac{\partial^2 T}{\partial x^2} \quad (5)$$

We can solve it for simplified situations



- u=0, Steady state $\left(\frac{\partial T}{\partial t} = 0\right)$

$$\kappa_f \frac{\partial^2 T}{\partial x^2} = 0 \quad (6)$$

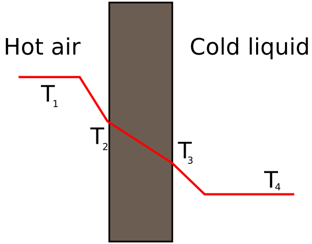
sol:

$$T(x) = \frac{T_{i+1} - T_i}{e_i}(x - x_i) + T_i \quad (7)$$

$$q(x) = \frac{T_i - T_{i+1}}{e_i/\kappa} \quad (8)$$

$$\rho_f c_{p,f} \frac{\partial T}{\partial t} + \rho c_p u \frac{\partial T}{\partial x} = \kappa_f \frac{\partial^2 T}{\partial x^2} \qquad (9)$$

However not for complex situations ($u \neq 0$) there is no analytical solution



Solution? To define global heat transfer coefficients

$$q = h(T_4 - T_1) \qquad (10)$$

we need to obtain expressions for h.

$$h = f(u, \kappa, \rho, c_p, geometry, ...)$$

It is better to define these formulas in terms of nondimensional numbers

Procedure:

1. Identify type of flow
    - Turbulent/laminar
    - Single phase/Multiple phase (determine regime)
2. Chose the aproppiate correlation for the heat/mass transfer coefficients

In industrial applications we have flows inside pipes. How to characterize the flow?

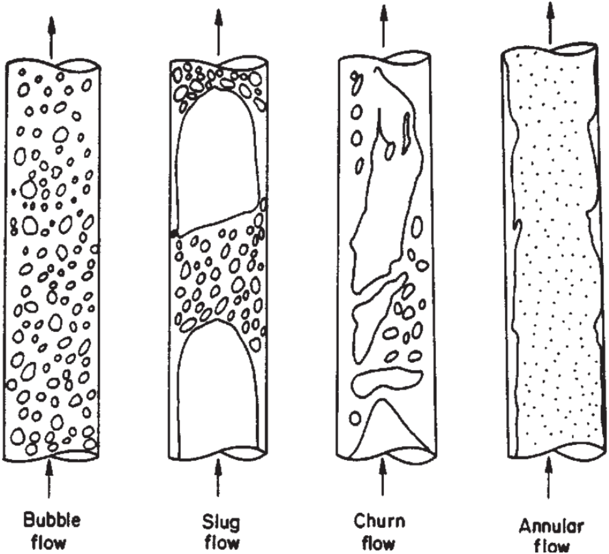


Figure: Flow types in vertical tubes

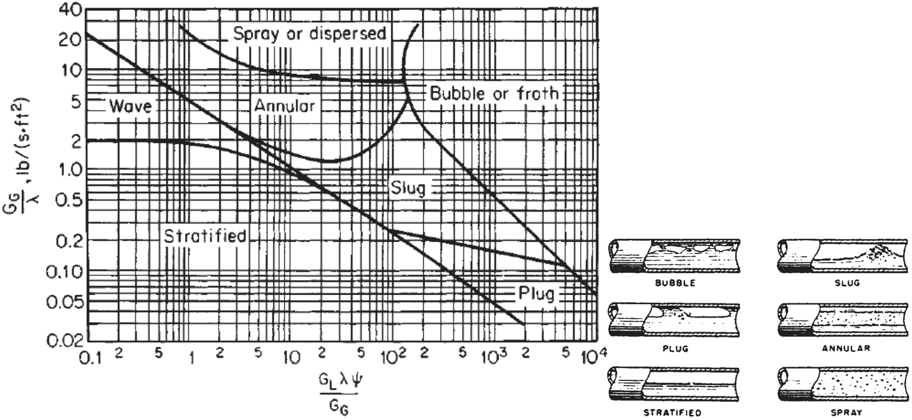


Figure: Flow pattern regions in cocurrent liquid/gas flow through horizontal pipes

where G_G is the gas mass velocity and $\lambda = \sqrt{\frac{\rho_G}{\rho_{air}} \frac{\rho_L}{\rho_{water}}}$ and

$$\Psi = \frac{\sigma_{water}}{\sigma} \left(\frac{\mu_l}{\mu} \left(\frac{\rho_{water}}{\rho_L} \right)^2 \right)^{1/3}$$

- Reynolds: Transition from laminar to turbulent flows

$$\mathrm{Re} = \frac{\rho U L}{\mu} \tag{11}$$

- Nusselt: Related to heat transfer. Convection/Conduction

$$\mathrm{Nu} = \frac{hL}{\kappa} \tag{12}$$

  Turbulent $\mathrm{Nu} \gg 1$
- Prandtl number: Momentum diffusivity to thermal diffusivity ratio

$$Pr = \frac{\nu}{\alpha} = \frac{c_p \mu}{\kappa} \tag{13}$$

- Pecklet number: Advection/Diffusion

$$\mathrm{Pe} = \frac{LU}{\alpha} = \mathrm{RePr} \tag{14}$$

- Weber number: Inertial/surface tension

$$We = \frac{\rho U^2 L}{\sigma} \tag{15}$$

Some correlations for evaporation:

- Chen (vapor phase > liquid phase)

$$\mathrm{Nu} = \frac{hD}{\lambda_l} = 0.023\mathrm{Re}_l^{0.8}\mathrm{Pr}^{0.4}F \qquad (16)$$

  where F is called effective boiling convection factor (tabulated from 1 to 100).

  One must be very careful looking at the definitions used to derive each correlation!!

- Rohsenow (liquid phase > vapor phase)

$$\mathrm{Nu} = 0.006 - 0.015\mathrm{Re}^{2/3}\mathrm{Pr}^{-0.7} \qquad (17)$$

Correlations depend on the geometry of the problem (vertical/horizontal tubes, turbulent/laminar flow, ...)

Once we know h (W/m^2K), we can obtain the surface S from Q

$$Q = \int q dS = \int h(T_{in} - T_{out}) dS \quad (18)$$

The total surface will give us the length of the tube

$$S = \pi D L_{tube} \quad (19)$$

Remarks

- This can be applied to very different problems (e.g. infinite plates)
- The same applies for mass transfer (in this case we need to define a global mass transfer coefficient)

$$\frac{\partial \alpha}{\partial t} + u \frac{\partial \alpha}{\partial x} = D \frac{\partial^2 \alpha}{\partial x^2} \quad (20)$$

α: Mass fraction

Heterogeneous models:

- Fully resolved models
- Disperse systems: Typically modeled as Lagrangian Particles

Lagrangian particle advection

In principle, the equations to obtain the trajectory of the particles are simple

- Velocity:

$$m^i \frac{d\boldsymbol{v}^i}{dt} = \boldsymbol{F}_L + \boldsymbol{F}_D + \boldsymbol{F}_F + \boldsymbol{F}_I + \boldsymbol{F}_{ext}$$

- Position:

$$\frac{d\boldsymbol{x}^i}{dt} = \boldsymbol{v}^i$$

The challenge is to correctly model the forces acting on the particle.

Forces implemented in the code: ***Drag Force***:
Force induced by the fluid on the particle due to viscous effects

$$F_D = -\frac{3}{4d_i} C_D Re_{p,i} \rho V^i |\boldsymbol{v}^i - \boldsymbol{u}|(\boldsymbol{v}^i - \boldsymbol{u})$$

where C_D is a drag coefficient (must be given), $\boldsymbol{v}^i$ is the particle velocity and $Re_{p,i}$ is the Reynolds number based on the particle diameter.

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Forces implemented in the code: ***Inertial Force***:

Fictitious force appearing as a consequence of the frame of reference chosen to follow the particle

$$F_I = \rho V_i \left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} \right]$$

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- Position:

$$\frac{d\boldsymbol{x}^i}{dt} = \boldsymbol{v}^i$$

The challenge is to correctly model the forces acting on the particle.

Forces implemented in the code: ***Added Mass Force***:

Force representing the inertia force induce by the volume of fluid displaced by the particle in the subgrid scale

$$F_A = \rho V_i C_M \left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} - \frac{d\boldsymbol{v}^i}{dt} \right]$$

Lagrangian particle advection

In principle, the equations to obtain the trajectory of the particles are simple

- Velocity:

$$m^i \frac{d\boldsymbol{v}^i}{dt} = \boldsymbol{F}_L + \boldsymbol{F}_D + \boldsymbol{F}_F + \boldsymbol{F}_I + \boldsymbol{F}_{ext}$$

- Position:

$$\frac{d\boldsymbol{x}^i}{dt} = \boldsymbol{v}^i$$

The challenge is to correctly model the forces acting on the particle.

Forces implemented in the code: ***Lift Force***:

$$F_L = -\rho C_L V^i (\boldsymbol{v}^i - \boldsymbol{u}) \times \omega$$

Lagrangian particle advection

In principle, the equations to obtain the trajectory of the particles are simple

- Velocity:

$$m^i \frac{d\boldsymbol{v}^i}{dt} = \boldsymbol{F}_L + \boldsymbol{F}_D + \boldsymbol{F}_F + \boldsymbol{F}_I + \boldsymbol{F}_{ext}$$

- Position:

$$\frac{d\boldsymbol{x}^i}{dt} = \boldsymbol{v}^i$$

The challenge is to correctly model the forces acting on the particle.

Forces implemented in the code: ***External Force*** (e.g. Bouyancy):

$$F_B = (\rho_p - \rho)\boldsymbol{g}$$

In addition, we need extra equations to model the heat and mass transfer.
For bubbles, we need an equation for the dynamic of the bubbles